SPATIAL AND TEMPORAL BEHAVIOR OF THE WEST POINT SEWAGE EFFLUENT IN PUGET SOUND

Kenneth Richard Karr

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Abstract

Data obtained from a spatial and temporal distribution study of sewage effluent, traced with fluorescent dye, discharged from the Municipality of Metropolitan Seattle (METRO) West Point treatment facility on Puget Sound are presented.

Following an initial dilution by a factor of about 1/100 at the discharge line diffuser ports, effluent diluted further by only 1/10 was found approximately 6000 meters north (ebbing current) and 3500 meters south (flooding current). Significant concentrations diluted by a factor of about 1/3 after leaving the diffuser ports were found near shore, and concentrations diluted by a factor of 1/2.3 were found on the West Point beach (3.7 ppt effluent).

From contours of sewage concentration and density, wide spatial and temporal variations were observed in both throughout the 5 days that the experiments were conducted (30 August through 3 September 1974). Transverse cuts of the plumes indicated that contours of sewage effluent generally coincide with density contours indicating that density stratification is influential in confining the distribution of sewage in the Sound. A longitudinal cut, however, indicated significant penetration of density layers by the effluent near the ends of the effluent plume. Implications of these phenomenon

on modeling effluent diffusion in the Sound are discussed. A general buildup of dye in the Sound over the period of the experiments was not observed.

Comparison of these dye studies with drogue studies carried out simultaneously show good agreement quantitatively for the 30 August data and qualitatively for data obtained on other days.

I. Introduction

From 30 August to 3 September, 1974, the Applied Physics Laboratory (APL) of the University of Washington undertook a study of the sewage effluent distribution in Puget Sound discharged by the Municipality of Metropolitan Seattle (METRO) processing plant located on West Point, Seattle. This study addresses two principal questions: Where does the sewage go? What is the nature of the mixing process acting on the sewage as it is transported through Puget Sound? By observing the nature of the answers to these two questions, conclusions regarding the applicability of model types can be drawn. Whether a buildup of sewage concentration occurs or whether sewage residence times at detectable concentrations are limited is investigated as well.

Data were obtained by cycling a towed sensor, "fish," through depths where the sewage was detected. Fluorescent dye was added to the sewage effluent as it was discharged from the processing plant. This dye was detected by the fish as it was towed through the effluent in the Sound. On board, real-time instrumentation allowed accurate determination of dye concentration, fish depth, and vessel position with respect to the sewage effluent discharge ports (outfall). The effluent was traced from the outfall to the areas in the Sound that it reached. The fish was capable of determining temperature and conductivity. Density data determined from the tempera-

ture and conductivity data are also used in this analysis.

Dye concentration, density, depth and vessel location data were recorded on magnetic tape on board and then processed ashore to provide raw information used to hand contour the data. The visualizations permitted by the contoured data lead to most of the observations made in this report.

An investigation of current properties using drogue information was conducted by Evans-Hamilton, Inc. in parallel with the dye studies.[1] Comments concerning the conclusions reached by these studies are made as a result of the data obtained from the dye studies.

This investigator prepared several programs for use with the hand held computer-calculator, HP-65, for navigation and data work-up that are presented in Appendix H.

II. Experimental Procedure[2]

Rhodamine B dye was injected into the sewage effluent just upstream of the post-chlorinators at a constant rate of 1.73 kg/hr (dry weight) for the first four days of the experiment (30 August - 2 September). The flow from the outfall (located about 1100 meters west of West Point at a depth of 73 meters) was 70.9 million gallons/day (mgd) over this period yielding an average concentration of dye in the pipe of 1.5×10^{-7} g/cc. On 30 August the average flow was 82.8 mgd and the dye injection rate was 1.85 kg/hr yielding a concentration in the outfall of 1.4×10^{-7} g/cc. A dye injection rate six times that

for the first four days was planned for the fifth day, but cancelled due to towing cable failure.

Dye concentration was measured with an instrument package("fish") towed behind APL's 50-foot research boat and cycled over a depth appropriate for detecting the dye. Prior to pumping dye into the system, a background fluorescence level equivalent to 5×10^{-11} g/cc dye concentration was determined in Puget Sound waters. Since dye concentration in the outfall is about 1.5×10^{-7} g/cc, this means that a minimum effluent concentration of 0.33 parts per thousand (ppt) could be observed. This corresponds to a maximum dilution factor of 1/3000 observable. Towing speeds varied from 3 to 6 knots. The fish consists of temperature, electrical conductivity, dye concentration, and depth (pressure) sensors housed in a streamlined aluminum tube. A pump within the fish forces water through the sensors. An electronics package accepts sensor output, multiplexes the data onto a single conductor within the towing cable and transmits the information to shipboard equipment. Figure 1 shows the general configuration of the fish and internal equipment. A hydraulic winch was used to tow and cycle the fish vertically. This gasoline powered unit is capable of hauling in at a rate of 1 meter/second. Both water depth and fish depth indications were displayed at the operators station enabling the operator to cycle the fish near the bottom while avoiding a collision with it.

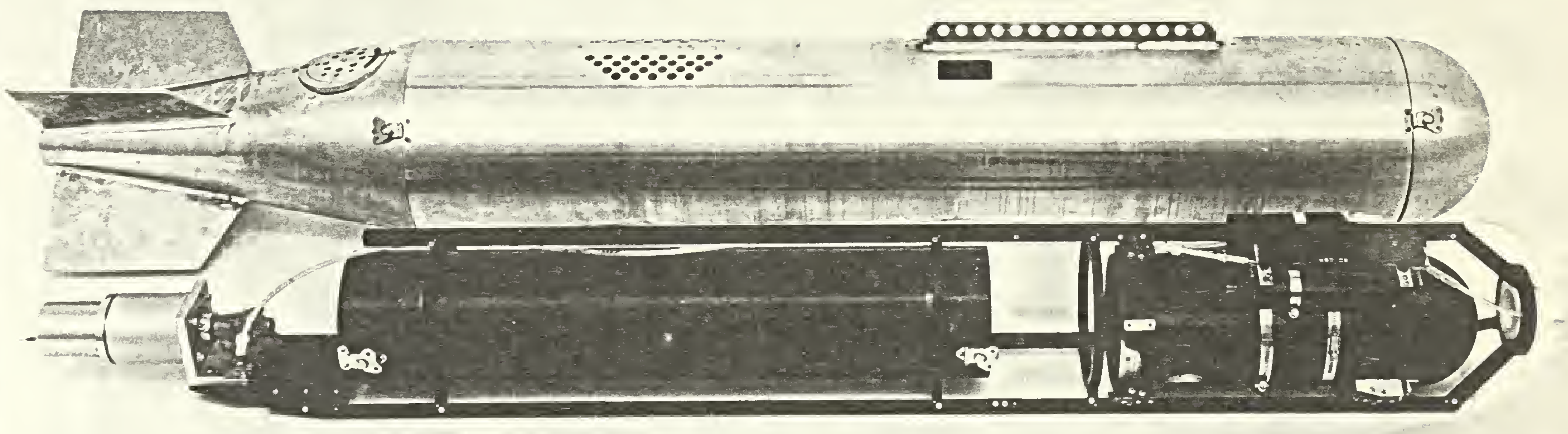


FIGURE 1

"FISH" AND INTERNAL EQUIPMENT

The boat's position was determined to within a few meters using a Motorola Mini-Ranger radar navigation system[3], which determines range between each of two on-shore transponder units and the shipboard master transmitting unit. Ranges were continuously displayed on the face of the master console aboard the boat. An HP-65 programmable calculator was used to convert range-range information to X-Y coordinates based on the sewage outfall position. This real time navigational information was essential to assure that the boat's navigation closely followed desired paths with respect to the outfall.

Ranges, dye concentration, fish and water depths, and clock time were all available aboard during the conduct of the experiment. Range, all fish data, water depth, time, and manually entered "event number" were recorded once per second on magnetic tape that is compatible with APL's CHI 2130 computer.

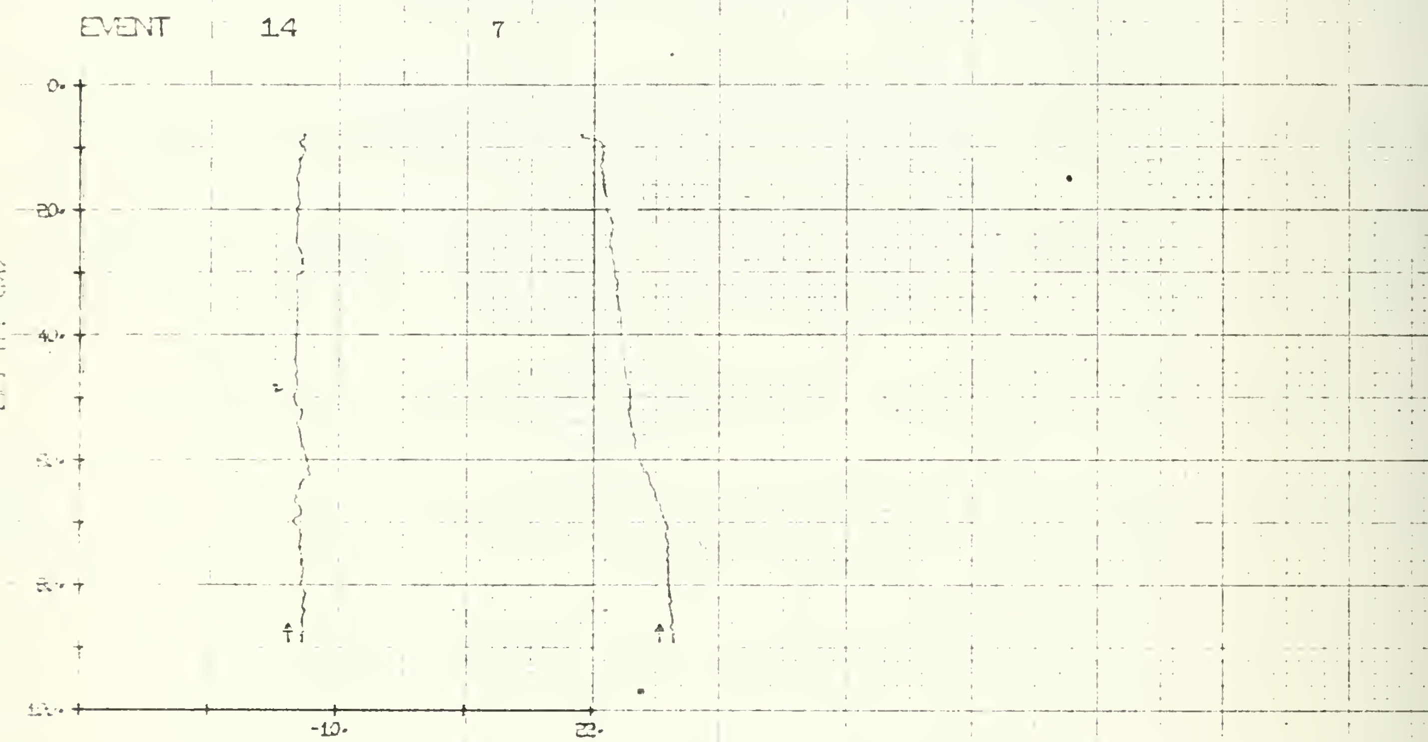
Prior to the experiment, a series of observations was made of the UW Oceanography Department's Puget Sound Model in order to provide a rough feeling for the expected behavior of the dye. A wide degree of variability from tidal cycle to tidal cycle suggested that a preset pattern for sampling would not be workable. Instead, it was decided that a series of arcs crossing the Sound in east-west directions starting near the outfall and working away in the direction of the current would be the most productive means for detecting the area containing the

effluent. In addition to following this general strategy, sweeps back up the plume(s) were conducted; and several excursions through water outside the boundaries of well defined plumes were made to detect detached pools that observations of the Puget Sound Model suggested might exist.

These studies and the drogue studies were coordinated in that the depth at which dye was detected was communicated to Evans-Hamilton personnel for use in setting drogue depth, and the establishment of flow following slack water was determined from the drogues and used to time the commencement of the dye detection sweeps. Normally, about 30 minutes were allowed to pass after flow commenced in order to assure a well established current pattern prior to sampling.

III. Dye Concentration and Density Contouring

Data obtained aboard on magnetic tapes was returned to the APL and processed on the CHI 2130 computer to plot dye concentration and density profiles for each cycle of the fish. Figure 2 is a sample of this output that was used in plotting dye and density contours. The boat's position at the beginning and end of each profile was determined from a printout of the data in tabular form as shown in Figure 3. Using this information, plots can be made that correspond to the X and Y positions the ship traversed. The tracks made during the dye detection sweeps, however, closely approximated straight lines.



DYE CONCENTRATION SCALE = 1 INCH/ 1. LOG UNITS BASE = -10. LOG UNITS

SIGMA-T SCALE = 1 INCH/ 1. SIGMA-T BASE = 22. SIGMA-T

FIGURE 2 - OUTPUT USED FOR PLOTTING DYE AND DENSITY CONTOURS

1	6	[illegible]	12.7[illegible]	-10.2[illegible]	131.[illegible]	0.	[illegible]	[illegible]	-10[illegible]	-[illegible]	[illegible]	[illegible]	0.0[illegible]
1	11	[illegible]	12.645	-10.2[illegible]	133.[illegible]	0.	10.[illegible]	[illegible]	-105.4	-212.[illegible]	[illegible]	21.87[illegible]	0.0[illegible]
1	16	4073030.	12.557	-10.234	33.4	0.	10.[illegible]	[illegible]	-107.5	-211.7	[illegible]	[illegible]	0.0[illegible]
1	21	[illegible]	12.536	-10.242	133.[illegible]	0.	11.42	[illegible]	-106.7	-215.[illegible]	23.[illegible]	21.7[illegible]	0.0[illegible]
1	26	[illegible]	12.602	-10.2[illegible]	1.6	0.	12.10	[illegible]	-107.8	-217.[illegible]	[illegible]	21.77[illegible]	0.0[illegible]
1	31	[illegible]	12.[illegible]	-10.2[illegible]	1.6	0.	13.[illegible]	[illegible]	-104.[illegible]	-214.[illegible]	[illegible]	[illegible]	0.0[illegible]
1	36	[illegible]	12.464	-10.251	1.5	0.	13.[illegible]	11.62	-105.5	-215.[illegible]	[illegible]	[illegible]	0.0[illegible]
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1	46	[illegible]	12.740	-10.227	1.7	0.	15.[illegible]	[illegible]	-106.6	[illegible]	[illegible]	[illegible]	0.0[illegible]
1	51	[illegible]	12.[illegible]	-10.231	1.7	0.	16.[illegible]	[illegible]	-101.0	[illegible]	[illegible]	[illegible]	0.0[illegible]
1	56	[illegible]	12.[illegible]	-10.237	1.7	0.	17.01	[illegible]	[illegible]	-21[illegible]	23.[illegible]	21.8[illegible]	0.0[illegible]
1	61	[illegible]	12.[illegible]	-10.235	1.6	0.	1[illegible]	[illegible]	-100.9	-22[illegible]	[illegible]	21.[illegible]	0.0[illegible]
1	66	[illegible]	12.[illegible]	-10.232	0.7	0.	19.40	[illegible]	-99.0	-22[illegible]	[illegible]	21.8[illegible]	0.0[illegible]
1	71	[illegible]	12.2[illegible]	-10.231	1.3	0.	20.[illegible]	[illegible]	-104.4	-217.[illegible]	23.[illegible]	21.8[illegible]	0.0[illegible]
1	76	[illegible]	12.2[illegible]	-10.2[illegible]	-23.[illegible]	0.	21.[illegible]	[illegible]	-10[illegible]	-218.7	23.[illegible]	[illegible]	0.0[illegible]
2	1	[illegible]	12.106	-10.2[illegible]	-23.4	0.	2[illegible]	[illegible]	-97.0	-2[illegible]	[illegible]	21.90[illegible]	0.0[illegible]
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2	11	[illegible]	12.075	-10.231	-166.[illegible]	0.	24.10	31.37	[illegible]	-2[illegible]	[illegible]	[illegible]	0.0341[illegible]
2	16	[illegible]	12.045	-10.231	-23.[illegible]	0.	24.[illegible]	[illegible]	-9[illegible]	-2[illegible]	[illegible]	[illegible]	0.0341[illegible]
2	21	[illegible]	12.043	-10.2[illegible]	-20.[illegible]	0.	24.1[illegible]	31.[illegible]	-9[illegible]	-241.[illegible]	23.[illegible]	[illegible]	0.0341[illegible]
2	26	[illegible]	12.045	-10.[illegible]	-21.[illegible]	0.	24.23	31.[illegible]	-94.1	-240.5	23.[illegible]	22.01[illegible]	0.03411
2	31	[illegible]	12.040	-10.2[illegible]	-22.[illegible]	0.	24.[illegible]	31.[illegible]	-9[illegible]	-241.[illegible]	23.[illegible]	22.01[illegible]	0.0341[illegible]
2	36	[illegible]	12.040	-10.2[illegible]	-21.[illegible]	0.	24.[illegible]	[illegible]	-94.1	-240.5	23.[illegible]	22.0[illegible]	0.0341[illegible]
2	41	[illegible]	12.035	-10.231	-19.3	0.	24.[illegible]	31.[illegible]	-9[illegible]	-231.0	23.[illegible]	22.0[illegible]	0.0341[illegible]
2	46	[illegible]	12.035	-10.233	-21.2	0.	24.[illegible]	31.27	-9[illegible]	-229.[illegible]	23.[illegible]	22.0[illegible]	0.0341[illegible]
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2	61	[illegible]	12.032	-10.243	-19.1	0.	24.1[illegible]	31.[illegible]	-97.2	-233.5	23.7[illegible]	22.0[illegible]	0.0341[illegible]
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3	36	[illegible]	12.[illegible]	-10.2[illegible]	-211.[illegible]	0.	1[illegible]	[illegible]	-101.5	-2[illegible]	23.[illegible]	21.[illegible]	0.0342[illegible]
3	41	[illegible]	12.247	-10.2[illegible]	-185.[illegible]	0.	1[illegible]	[illegible]	-10[illegible]	-2[illegible]	23.[illegible]	21.[illegible]	0.0342[illegible]
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3	51	[illegible]	12.430	-10.2[illegible]	-215.6	0.	8.0[illegible]	[illegible]	-7[illegible]	-251.5	[illegible]	21.6[illegible]	0.034[illegible]
3	56	[illegible]	12.713	-10.2[illegible]	-217.[illegible]	0.	5.[illegible]	[illegible]	-71.3	-2[illegible]	[illegible]	21.7[illegible]	0.0344[illegible]
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4	21	[illegible]	12.[illegible]	-10.2[illegible]	-22[illegible]	0.	[illegible]	[illegible]	[illegible]	-2[illegible]	[illegible]	21.[illegible]	0.0[illegible]
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4	31	[illegible]	12.[illegible]	-10.2[illegible]	-22[illegible]	0.	[illegible]	[illegible]	[illegible]	-2[illegible]	[illegible]	21.[illegible]	0.0[illegible]
4	36	[illegible]	12.[illegible]	-10.2[illegible]	-22[illegible]	0.	[illegible]	[illegible]	[illegible]	-2[illegible]	[illegible]	21.[illegible]	0.0[illegible]
4	41	[illegible]	12.[illegible]	-10.2[illegible]	-2[illegible]	0.	[illegible]	[illegible]	[illegible]	-2[illegible]	[illegible]	21.[illegible]	0.0[illegible]
4	46	[illegible]	12.[illegible]	-10.2[illegible]	-2[illegible]	0.	[illegible]	[illegible]	-1[illegible]	-2[illegible]	[illegible]	21.[illegible]	0.0[illegible]
4	51	[illegible]	12.[illegible]	-10.2[illegible]	-2[illegible]	0.	[illegible]	[illegible]	-1[illegible]	[illegible]	[illegible]	21.[illegible]	0.0[illegible]
4	56	[illegible]	12.[illegible]	-10.2[illegible]	-2[illegible]	0.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	21.[illegible]	0.0[illegible]
4	61	[illegible]	12.[illegible]	-10.234	-23[illegible]	0.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	21.[illegible]	0.0[illegible]

FIGURE 3 - TABULAR DATA OUTPUT

This allowed simplifying the contoured data to be plotted with respect to distance along the approximate straight line track. The straight line tracks are shown as the first figure of each of Appendices A through F. Each plot that follows in these Appendices shows distance along the respective tracks (corresponding to the appropriate event numbers) with the zero point corresponding to X = 0 or Y = 0 depending upon whether the slope of the track lies within 45° of the X or the Y axis respectively. Where density contours have been plotted, they were done on translucent paper so that they may be overlaid on the corresponding dye contours. This method greatly improved the visualization of the contoured data and leads to some of the conclusions expressed in this report. The depth dimension used in each case is 10 meters/inch with the water surface at the top line. All contoured data are presented as though the observer were looking north for the east-west cuts across the plumes, and as though the observer were looking west for the north-south cuts. Each data point on both the dye concentration and density contours is indicated by a symbol that represents the observed concentration at the point plotted. Figure 4 provides a listing of all symbols used.

The reader will note upon a cursory inspection of the plotted data that it is closely spaced along the path of the fish compared to the horizontal distance between successive depth cycles. During the preliminary

Figure 4

Contour Plot Symbols

Symbol	Dye Concentration (g/cc)	Density (σ_t)
/	$10^{-10.4}$	21.8
–	$10^{-10.3}$	21.9
·	$10^{-10.2}$	22.0
x	$10^{-10.1}$	22.1
o	$10^{-10.0}$	22.2
△	$10^{-9.9}$	22.3
□	$10^{-9.8}$	22.4
*	$10^{-9.7}$	22.5
◇	$10^{-9.6}$	22.6
◖	$10^{-9.5}$	22.7
⊙	$10^{-9.4}$	
⊗	$10^{-9.3}$	
◎	$10^{-9.2}$	
▽	$10^{-9.1}$	
⌓	$10^{-9.0}$	
+	$10^{-8.9}$	

stages of preparing the data, computer programs that were capable of contouring were investigated and it was found that none were capable of handling data that was so unevenly distributed. Therefore, hand contouring became necessary. Although, in most cases, contours could be drawn in only one way, occasionally, the data became ambiguous and a judgement regarding the most appropriate way to draw the contour had to be made.

The contoured data are presented in six segments in Appendices A through F. These correspond to the data obtained on the sweeps indicated in Table 1.

Table 1

Experiment Descriptions

Day	Date	Flood/ Ebb	North/ South of Outfall	Direction of Sweep (Toward or Away from Outfall)	Appendix
1	30 Aug.	Flood	South	Away	A
2	31 Aug.	Ebb	North	Away*	B
2	31 Aug.	Flood	South	Away	C
2	31 Aug.	Flood	South	Toward	D
4	2 Sep.	Ebb	North	Single loop made in area	E
5	3 Sep.	Ebb	North	Single loop made in area	F

* The last event of this segment was a sweep back up the plume toward the outfall.

The time each new event was begun is indicated on

each track segment on the track plots. The reader will note the significant time spans encompassed in the patterns presented. It is obvious that the data presented is not synoptic. Over the period encompassed during single events, the contours probably do approximate a synoptic picture in that during the short period of time (about 30 minutes more or less) the pattern existing in the Sound is unlikely to be changed very much. Furthermore, changes that do take place within this time frame are likely to conform to the random processes which produce a given pattern that exists at any given instant. The same statement is not observed to apply over time scales commensurate with that of the tidal cycle itself. Examples of significant change in the plume patterns are found in the data during later sweeps of the plume area within the same tidal cycle.

The times of the slack waters that preceded and followed the plotted tracks, as determined from tidal current tables, are also noted on each track plot.

IV. Observations

The data, as presented in Appendices A through F, are intended to be useful to the reader for the purposes of whatever perspective might be appropriate to his interests. Pertaining to the specific objectives of the METRO study, however, some observations are offered.

Variability of Sewage Concentration and Density

The immediate and obvious observation upon inspection of the contoured data is the spatial variability of both dye concentration and density that was encountered. Of all the data obtained, day 1 contours are the most uniform, but even these show significant variations in depth and along the track. Some of these events show two areas of high dye concentration instead of a single, uniformly developed peak (see events 7 and 10.2, day 1) that might be expected of an "idealized" plume. Intrusions of clean water (concentration of dye less than background) are seen to divide the plume in depth also (see events 11, 12, and 13, day 1).

The data obtained on days 2, 4, and 5 all show patterns of dye concentration more complex than that of day 1. Four or more discrete filaments are discernible in much of this data, providing an indication of the complex mixing processes occuring in the Sound. It bears mentioning, again, that caution must be exercised in analyzing the contoured data presented since a significant amount of time passes from the start to the completion of each event.

Models and Implications

The foregoing observations bear significant impact on the types of models that may be devised to describe the effluent distribution in the Sound. Many models assume that the diffusion is radially symmetric, and,

further, that probability distributions are Gaussian or of a shape similar to that of a Gaussian distribution. From the foregoing observations, it is evident that neither of these conditions hold in the Sound. The day 1 data most closely approximates these conditions, but even this data implies that such models must be used in a very limited fashion. It has been noted, however, that although the distributions observed hardly conform to the conditions mentioned above, a good fit was obtained on day 1 for a model that assumes the "centerline" concentration to be approximately an inverse function of distance from the outfall (concentration proportional to $1/Y^a$, where $1 \leqslant a \leqslant 3/2$)[4]. The data from day 4 appear to conform to such a model as well, but only 3 events were obtained across the plume(s), and the data is, therefore, not conclusive. Certainly, any model that attempts to span a time scale greater than one tide could not successfully incorporate the assumptions of a smooth distribution and radial symmetry.

A further assumption, that the effluent may be described in two horizontal dimensions, is frequently made in models used to describe flows such as under study here. In order for such an assumption to be valid, sufficient stratification must be present to confine the effluent flow along density contours only. It will be seen in the following observations that, although stratification greatly affects the distribution of effluent, this assumption,

also, is not valid during portions of the tidal cycle.

Depth Distribution of Effluent

From the overlays of density contours it is seen that the stratification in the Sound is influential in the observed distribution of sewage as indicated by the dye. Little evidence of dye contours crossing density contours exists in the east-west crossings of the plume(s) on day 1. Subsequently, on day 2 and 4 data for instance, the trend for dye contours to follow density remains evident, although frequent crossings do occur. This may be partially due to the method in which the data was processed. Following the first day's experiment, a significant lag in density detector(s) response occurred, possibly due to a pump or piping malfunction (leak) that went undetected. This necessitated averaging the density data between successive half cycles to eliminate the time lag. Therefore, the contours presented for days 2 and 4 represent values averaged over from 50 to 200 meters, depending upon the distance covered by the boat between cycles. The smaller scale density fluctuations were thus averaged out while the dye detection equipment continued to function properly, allowing the smaller scale dye concentration structure to be observed.

It is noteworthy that, in areas very close to the outfall and at low current velocities, the effluent was able to significantly affect the density structure. Event 5 on day 1, event 12.1 on day 2, and event 2 on

day 4 all show a divergence of density contours surrounding areas of high dye (sewage) concentration. These areas may be considered "active" in the sense that the deeper portions of the high sewage concentrations are lighter than their surroundings and the shallower portions are heavier. These conditions were generally observed within about 400 meters or less of the outfall. During periods near the outfall when the current velocity is relatively low, significant vertical penetration of the dye appears to take place from these active processes and may lead to the observations described in the following paragraphs.

Longitudinal Distribution of Sewage

Event 11 on day 2 is a longitudinal track from north to south near the end of an ebbing current. The density contours overlaid upon the dye contours clearly show that dye penetrates density contours. The plume, shown to be separated by about 1400 meters in the portion generated during the latter half of the tidal cycle, displays ends in each segment that reach a lesser density layer at the northern end than is the case with the remainder of the segment. Because the density contours are penetrated, models that attempt a two dimensional description cannot be directly applied. (It has already been shown that a simple plume description is not appropriate due to the extreme variability of the flow both spatially and temporally.) The cause for this observed penetration of the density layers cannot be stated with much confi-

dence because of the limited amount of data available longitudinally. Because shallow portions of the plume segments lie at the ends, it is possible that the greater penetration occurs during periods of low current velocity that occurs during a change of tide and due to changing current conditions in the middle of a tidal cycle (perhaps the disestablishment of an inshore eddy significantly slows the current flow over the outfall for a period of time followed by a return to greater velocities as the new trajectory becomes established). During this period, lateral mixing processes might be more severely restricted and thereby diffuse less of the buoyant momentum of the effluent as it leaves the mixing ports of the outfall. This would tend to cause the effluent to penetrate to a shallower density area than if the current velocity were higher. Further investigation is needed to better clarify the processes that cause this phenomenon.

Surface Penetration

It is significant to note that no dye was detected on the surface during these experiments. In general, only infrequently did dye penetrate to above 20 meters, reaching only about 4 or 5 meters at the most. A significant exception to this occurred on day 5, from 2000 to 3500 meters north of the outfall. Significant concentrations were detected to about 4 meters of the surface, and it is likely from observations of the contoured data of events 0 and 2.2 of day 5, that dye may have reached

a depth of 1 or 2 meters. Significantly, wind conditions that prevailed during day 5 were different (though not particularly severe) than on the previous days that the experiments were conducted. Wind records from the West Point Coast Guard station show a reversal of the wind from NE on day 4 to SSE on day 5 with an increase in average speed from 7.5 to 10.7 knots. Although the magnitude of this speed increase does not seem large, a significant change of surface wave conditions were noted on day 5. Also, a lesser density stratification was noted. These conditions obviously had an impact on the dye distribution that was observed. It would be expected that an even greater distribution in depth might be noted in the winter, and this, from preliminary indications thus far in the winter experiment analysis, was the case.

Dilution of Sewage Effluent in the Sound

The maximum concentration of dye observed throughout the conduct of the experiments occurred during a flood tide on day 2, event 12.1. A value of 1.26×10^{-9} g/cc was observed 400 meters south and 200 meters east of the outfall. This represents a dilution factor of 1/119 since the mean concentration in the outfall on day 2 was 1.5×10^{-7} g/cc. Of interest is that a concentration of 1.6×10^{-10} g/cc was observed over 6000 meters north of the outfall on event 10, day 2. This represents a dilution factor of only 1/7.9 in the distance! Since the subsequent event, 11 on day 2, shows two segments,

the impression that the northerly segment belongs to a previous tidal cycle was gained, but found not likely. Direct measurement of current velocity by drogues was not accomplished on this day's experiment. However, by using the maximum current observed for all of the drogue studies (day 1), of 38 cm/sec, and the fact that the average current during a tidal current of the duration observed is approximately .64 times the maximum velocity, a current velocity average much in excess of 24.2 cm/sec over the tidal cycle involved is not likely. In six hours at this speed, a distance of 5230 meters could have been traveled by the leading edge of the plume without taking into consideration mixing that occurs. It is not unreasonable, therefore, that it reached 6000 meters in a single tidal ebb current. Possibly, during the portion of the ebb that occurred at relatively high current velocities, the plume was directed toward the eastern part of the Sound (toward Shilshole Bay) and as the ebb began and also near (but not at) the end when it slowed, it changed to a more central route up the center of the Sound. Such a pattern (and the many others that represent such widely variable characteristics) further serves to emphasize the difficulty in modeling flow in the Sound.

From these observations, it is evident that, as pointed out in the preliminary report to METRO, most of the mixing and dilution of the effluent occurs at the

diffuser ports at the outfall.[5] High concentrations were observed south of the outfall as well, although at a distance only half that observed to the north. On day 2, event 20, a concentration of 4.0×10^{-10} g/cc was observed nearly 3000 meters to the south and very near shore. This concentration represents a dilution factor of only 1/3.8 to the highest concentration observed near the outfall on this day.

The data indicates that sewage reaches considerable distances north and south of the outfall and very near shore with little dilution from mixing in the Sound, within one tidal cycle.

Buildup of Sewage

Throughout the five days the experiments were conducted, a buildup of a dye background was not observed. All dye observed showed distinct boundaries, whether part of the main plume(s) or drifting patches. The concentrations within the plume areas remained relatively consistent and clean (less than background dye concentration) water can be found bounding and intruding within the main concentrations. Several excursions were made outside the main plume(s) and dye was not detected. It is difficult to positively identify patches of dye as belonging to a previous tidal cycle, although event 4 on day 2 is a likely candidate since the location and concentration of this dye patch seems to indicate that it is not part of the main plume(s). It appears that

most of the sewage that enters the Sound may be mixed to below background concentrations within two tidal cycles.

Sewage Near Shore

Concentrations of effluent ranging as high as 2.7 ppt were observed on several occasions very near shore, both north and south of the outfall. This is somewhat evident during the day 1 experiment in events 7 and 11, but most vivid on day 2 during events 20 and 21 taken during southerly sweeps. Observations during northerly sweeps include day 4 event 3.1, and day 5, events 0 and 2.2. As shore is neared, water depth decreases and the depth to which the fish is cycled is correspondingly reduced and this is seen in the plots of the above tracks. Where the fish depth decreases in these plots significant concentrations of dye were detected, and, in most cases, the concentrations actually increase, possibly due to a near shore eddy. That effluent reaches shore is confirmed by bottle samples taken on both the north and south beaches of West Point. Dye concentrations as high as 5.1 $\times 10^{-10}$ g/cc (corresponding to an effluent concentration of 3.4 ppt) were observed on the beach north of West Point and as high as 5.5 $\times 10^{-10}$ g/cc (3.7 ppt effluent) south. It is evident that sewage diluted by a factor of 1/2.3 of that found to be the maximum after exiting the diffuser ports finds its way ashore.

V. Amount of Dye in the System

In an attempt to determine whether significant por-

tions of the effluent discharged into the Sound were going undetected, a mass balance of day 1 data for events 10.2 through 12 was made. Day 1 was chosen since the plume pattern was more uniform that day and a good cross section of the dye in the plume was obtained for each of the events used. Dye contoured data was averaged for each 5 m by 25 m area transverse to the plume. These values were then added to provide a figure for the total amount of dye per longitudinal centimeter as demonstrated by the following relation:

$$c_{ev}[g/cm] = \left(\sum_{i=1}^{n} C_i\left[\frac{g/cc}{25 \times 10^4\ cm^2}\right]\right) 25 \times 10^4 [cm^2]\ n$$

where: C_i = the concentration observed in an individual 1.25×10^6 cm^2 cross sectional area ($g/cm^3/1.25 \times 10^6$ cm^2)

c_{ev} = a linear concentration along the plume axis (g/cm) for a given event (ev)

n = the number of 1.25×10^6 cm^2 squares for which a concentration (C_i) was observed during the event

The values of c_{ev} for the three events were averaged, thereby entertaining the approximation that current properties were the same as each of these segments of water passed the outfall. Using the average value of $\overline{c_{ev}}$ = 3.08×10^{-2} (g/cm), and assuming that this average figure persisted for one hour (t = 3600 sec), and further

assuming that this segment of dye passed over the outfall at an average of one hour following the commencement of the flood tide (an approximation based on the time of placement of the drogues into the water and the observed position of the boat with respect to the drogues during this portion of the experiment) at a velocity half[6] of the maximum observed from the drogue data ($v = .5 \times 38$ cm/sec), the quantity of dye, q g/hr, entering the Sound is determined as follows:

$$\begin{aligned} q &= \overline{c_{ev}} \times v \times t \\ &= (3.08 \times 10^{-2})(0.5 \times 38)(3600) \\ &= 2100 \text{ g/hr (or 2.1 kg/hr)} \end{aligned}$$

The actual injection rate measured at the dye pumping equipment located at the West Point sewage treatment facility was 1.73 kg/hr. Agreement between these two figures is good and there is no indication that dye is entering the system and going undetected into areas outside the observed plume.

VI. Comparisons with Drogue Studies.

The data obtained on day 1 were digitized and analyzed to provide statistics that could be directly compared with those obtained by C. C. Ebbesmeyer of Evans-Hamilton, Inc. Density contours for events 6 through 14 of day one were used to prepare digitized data for each 5 m by 25 m square cross-section (1.25×10^6 cm^2). These

data are presented in Appendix G. Values shown are in tenths of a power of ten that is added to the power -10.3 of ten. For example; a value of 4.5 shown on the grid is equivalent to a power of 0.45 to be added to -10.3, yielding a power of ten equal to -9.85. The concentration represented is therefore $10^{-9.85}$ in this case, or 0.14×10^{-9} over the 1.25×10^{6} cm^2 area. The abscissa dimensions are arbitrarily selected digits each representing 25 meters along the track. A program was prepared for the HP-65 computer-calculator to calculate centroids in both dimensions, and another prepared to calculate the standard deviations and variances of the distributions. These programs are included as Appendix H.

The data determined in this way are presented as Table 2. A plot of a plume that corresponds to Figure 3a of the Evans-Hamilton final drogue report[7] is presented as Figure 5. Standard deviation data were converted to represent those obtained by C. C. Ebbesmeyer along the axis shown in his Figure 7a. A plot of this data versus distance from the outfall is presented as Figure 6, and can be directly compared with Figure 8 of the Evans-Hamilton report that is enclosed as Figure 7 (overlay Figure 6 on 7 for comparison).

The day 1 dye experiment analyzed in this way generally confirms the drogue data. Differences that do occur are probably more a result of the difference in time that the dye data was obtained with respect to that

Table 2

Day 1 Dye Statistics

Event No.	$\bar{X}$ (m)	$\bar{Y}$ (m)	$(10^{4}\;^{x}\,\mathrm{cm})$	$(10^{2}\;^{y}\,\mathrm{cm})$	$(10^{8}\;^{x}\,\mathrm{cm})$	$(10^{6}\;^{y}\,\mathrm{cm})$
6	188	53.5	1.99	10.8	3.96	1.18
7	183	57.2	2.21	12.7	4.91	1.62
10(1)	160	50.6	2.49	10.0	6.21	1.00
10(2)	359	54.0	2.65	15.0	7.03	2.25
11	188	57.3	2.42	14.7	5.87	2.17
12	294	54.9	2.40	16.2	5.78	2.61
13	411	59.6	3.78	14.9	14.31	2.21
14	315	56.4	4.77	13.9	22.73	1.93

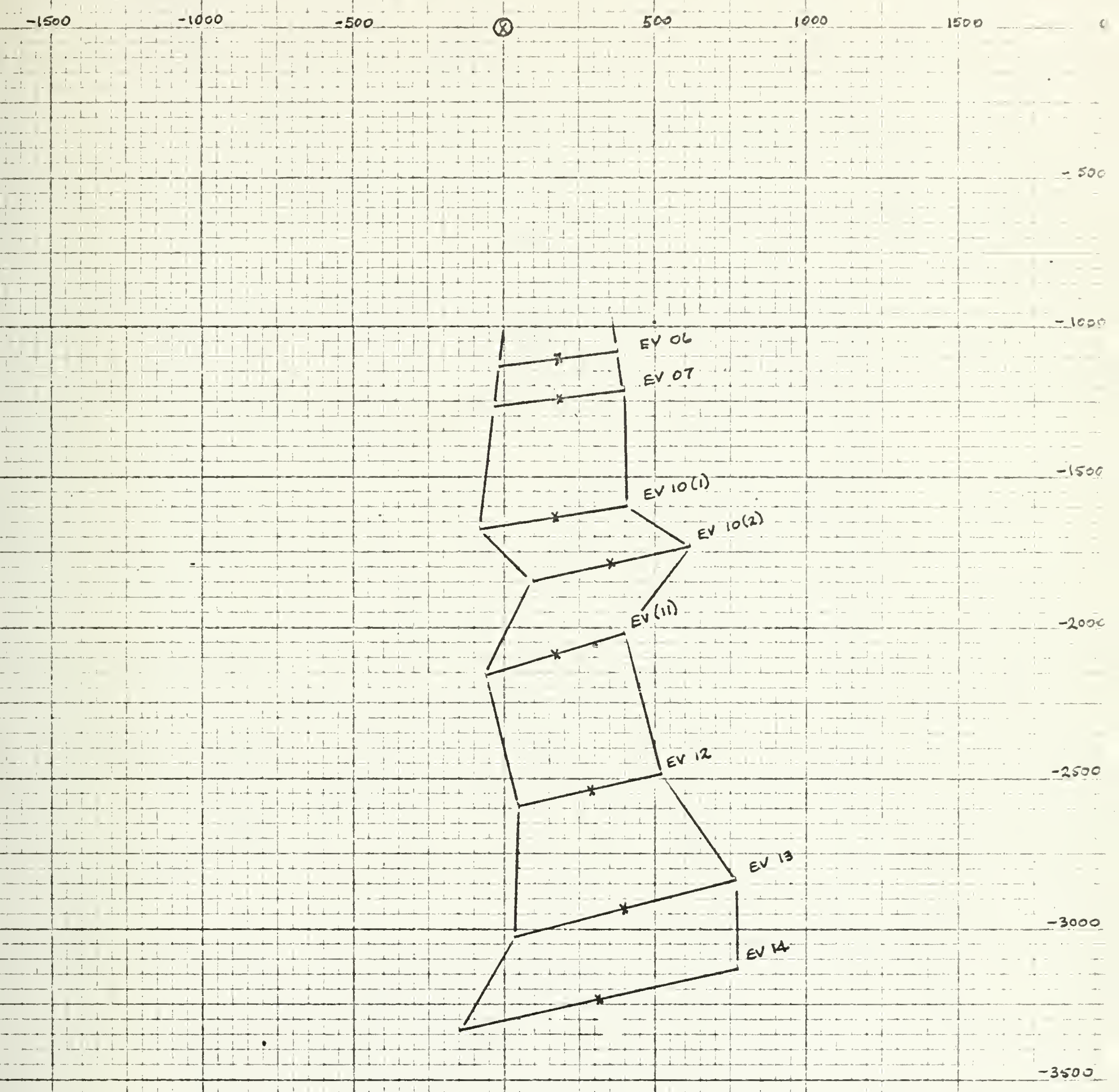


FIGURE 5

SIMULATED PLUME DETERMINED FROM DYE CONCENTRATION STATISTICAL DATA, 2 STANDARD DEVIATIONS IN WIDTH CENTERED ABOUT THE CENTROID (X)

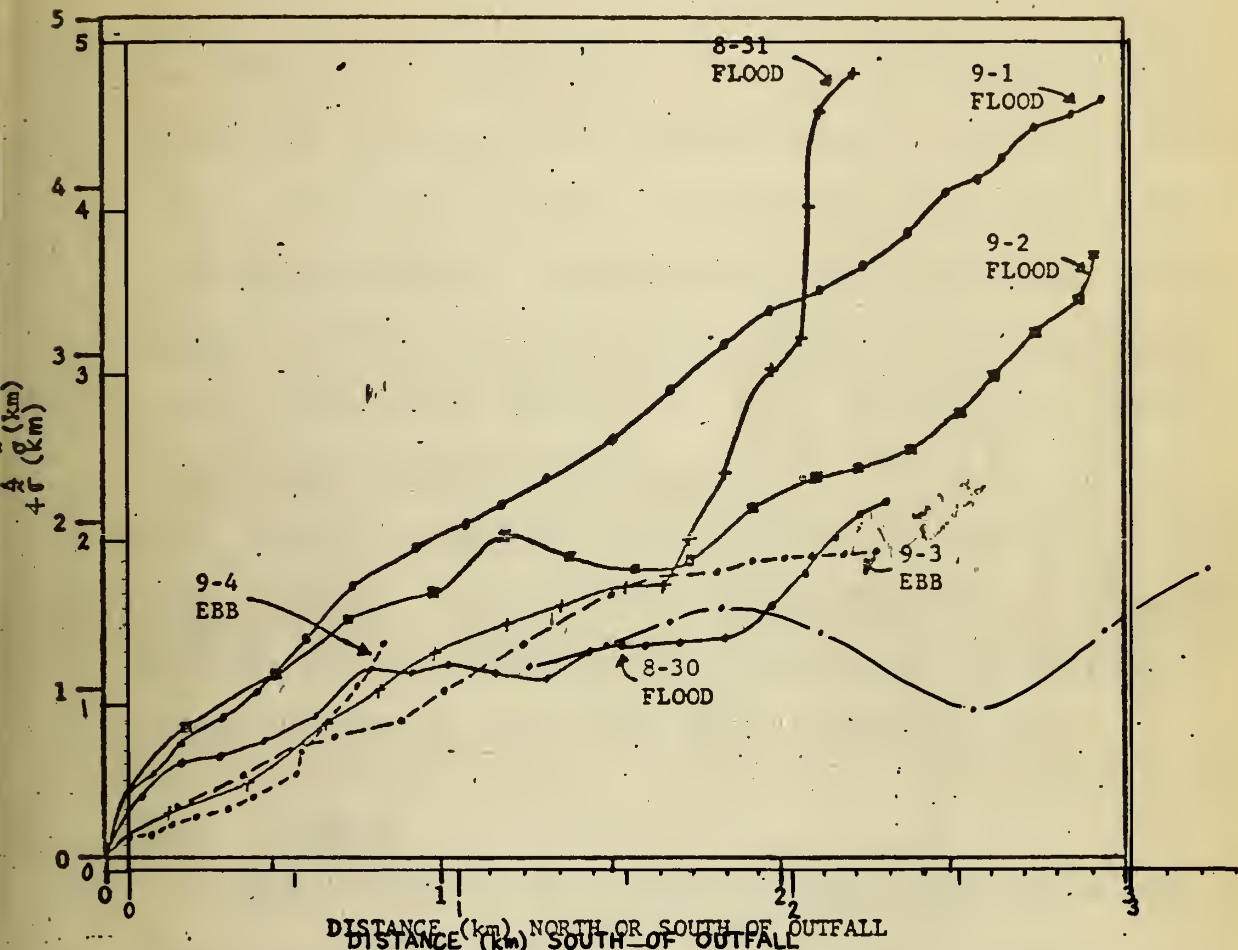


Figure 8. Average standard deviation (x4) of simulated plumes versus distance from outfall

FIGURE 6
AVERAGE STANDARD DEVIATION (x 4) OF PLUME (DAY 1)
VERSUS DISTANCE FROM OUTFALL

Figure 7

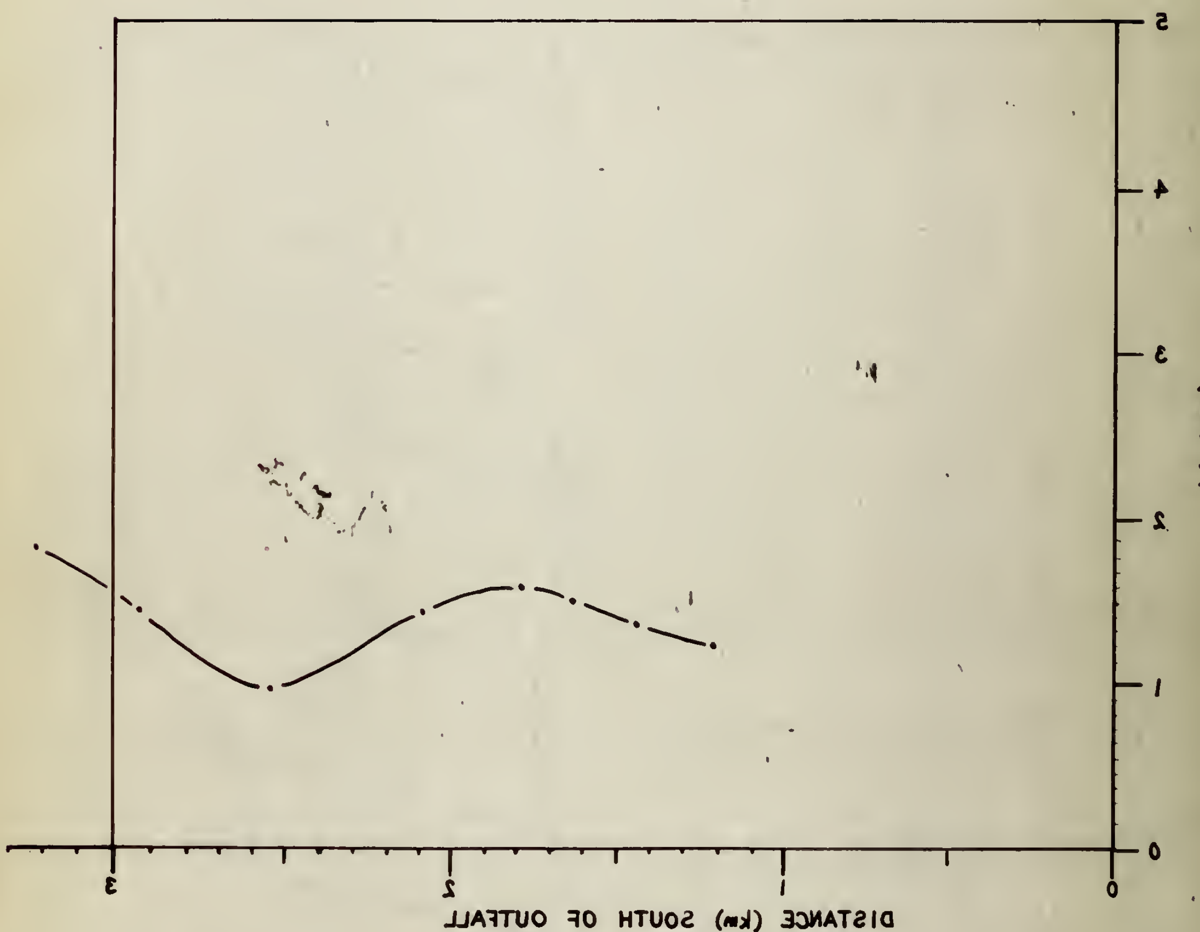


FIGURE 6
AVERAGE STANDARD DEVIATION (x 4) OF PLUME (DAY 1)
VERSUS DISTANCE FROM OUTFALL

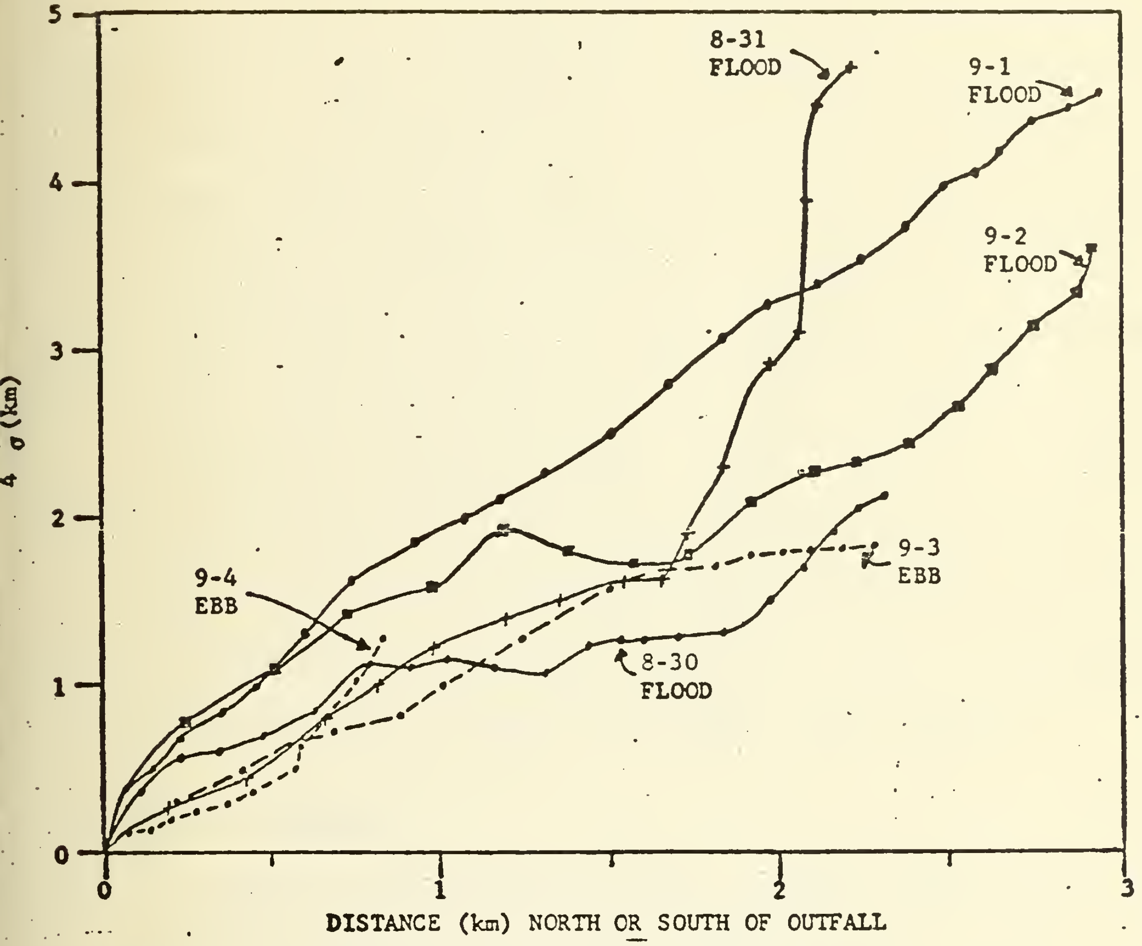


Figure 8. Average standard deviation (x4) of simulated plumes versus distance from outfall.

Figure 7

of the drogue data (dye data is about ½ to 1 hour later than the drogue data on day 1). Also, a qualitative inspection of data from experiments conducted on subsequent days confirms the general pattern observed using the drogues. However, from the dye studies, it is seen that distinct filaments are observed along with a wide degree of variability of concentration in general, and these observations would indicate that the current properties as observed using drogue data are likely to be significantly optimistic with respect to the concentrations that they predict. It is not possible to detect individual filaments using drogues, and it is evident that multi-filament "plumes" persist throughout the data. Eddy coefficients therefore cannot generally be predicted confidently using models consistent with the smoothly averaged type of data obtainable from drogues.

VII. Summary

Most of the mixing over a single tidal cycle apparently takes place at the diffuser port at the outfall. The maximum concentration found in the vicinity of the outfall was 1.26×10^{-9} g/cc, representing a dilution of the sewage by a factor of 1/119. After leaving the vicinity of the outfall, very large spatial and temporal variations of sewage concentrations are found in the Sound. (The same can be said of density profiles.) Indications of sewage diluted by a factor of less than

1/10 in the Sound proper were found as far as 6 km north and 3.5 km south of the outfall. Near shore, concentrations as high as 4×10^{-10} g/cc were noted, representing a dilution by a factor of 1/3.1 since entering the Sound. Sample bottles ashore were found to contain concentrations as high as 5.5×10^{-10} g/cc, representing a dilution by a factor of just 1/2.3.

The data indicate that the density stratification is influential in confining the distribution of sewage in depth, and little significant crossing of dye and density contours occurs in thè transverse cuts across the plumes. A longitudinal cut back up a plume near the end of an ebb current, however, demonstrated significant crossing of density contours by dye concentration contours which was most pronounced near the ends of the plumes. This may indicate that the sewage is more effective in penetrating the pycnocline over the outfall during low current velocities.

Models that are used to describe diffusion of the nature occuring from the METRO sewage outfall at West Point often assume that the flow is two dimensional, or even that the diffusion is radially symmetric. Experimental observations indicate that none of these assumptions can be applied to the conditions existing in the Sound without accepting very significant error except over an extremely limited range of space and time scales. If two-dimensional models are employed, they should be ap-

plied to that portion of the tidal cycles that contain well established currents, since some evidence exists that sewage penetrates density contours more during the initial stages of the cycle. A model that assumes the "centerline" concentration to be approximately an inverse function of distance from the outfall was applied to the data with limited success. Such a model applies during tidal cycles that produce a plume that is well defined and contains little transverse variation. Such a condition leads to less mixing than in plumes containing more variability (ie: more filaments or eddies). Only the data from day 1 provided a significant indication for success of such a model.

The buildup of a general dye background was not observed although it is not possible to say that the clear water observed in the experiment was a result of advection of dye out of the observation range (unlikely), or a result of dye having been diluted to below the limits of detectability. This experiment did not address the question of what happens after a tidal cycle is complete with respect to whether dye is transported back across the outfall, thereby surviving the current reversal. The winter experiment, under analysis at this time, indicates that significant patches of dye do, in fact, survive the reversal in current and are transported back across the outfall.[8]

Comparison of the dye studies data with drogue stu-

dies data indicates that the two are generally consistent. Significantly, however, the variability of conditions that were found in the dye studies cannot be determined by drogue studies. Therefore, the drogue data probably underestimates the peak concentrations likely to be observed in nature.

References

1. C. C. Ebbesmeyer and Akira Okubo, A Study of Current Properties and Mixing Using Drogue Movements Observed Near West Point in Puget Sound, Washington, Evans-Hamilton, Inc., Seattle, Washington, (November 1974), report prepared for the Municipality of Metropolitan Seattle, Seattle, Washington.

2. William P. Bendiner, Dispersion of Effluent from the West Point Outfall Interim Report, Applied Physics Laboratory, University of Washington, (15 January 1975), report prepared for the Municipality of Metropolitan Seattle, Seattle, Washington, pp. 1-4.
   The experimental procedure outlined in this report closely parallels the referenced document.

3. Detailed information concerning the Motorola Mini-Ranger III system used in this experiment may be obtained from Mr. Ernest H. Linger of the Applied Physics Laboratory, University of Washington, Seattle.

4. Bendiner, op. cit., p. 5.

5. Ibid., p. 6.

6. U. S. Department of Commerce, National Oceanic and Atmospheric Administration, Tidal Current Tables 1974, Rockville, Md.: National Ocean Survey, 1973, p. 232.
   Water passing over the outfall one hour after the commencement of the flood tide possessing an interval between slack and maximum current of 3 hours has a velocity .5 of the maximum current according to Table A.

7. Ebbesmeyer, op. cit., p. 5.

8. Personal communication with Mr. William P. Bendiner of the Applied Physics Laboratory, University of Washington, Seattle, March 31, 1975.

Appendix A

Day	Date	Flood/ Ebb	North/ South of Outfall	Direction of Sweep (Toward or Away from Outfall)
1	30 Aug.	Flood	South	Away

Appendix B

Day	Date	Flood/ Ebb	North/ South of Outfall	Direction of Sweep (Toward or Away from Outfall)
2	31 Aug.	Ebb	North	Away*

* The last event of this segment was a sweep back up the plume toward the outfall.

Appendix C

Day	Date	Flood/ Ebb	North/ South of Outfall	Direction of Sweep (Toward or Away from Outfall)
2	31 Aug.	Flood	South	Away

Appendix D

Day	Date	Flood/ Ebb	North/ South of Outfall	Direction of Sweep (Toward or Away from Outfall)
2	31 Aug.	Flood	South	Toward

Appendix E

Day	Date	Flood/ Ebb	North/ South of Outfall	Direction of Sweep (Toward or Away from Outfall
4	2 Sep.	Ebb	North	Single loop made in area

Appendix F

Day	Date	Flood/ Ebb	North/ South of Outfall	Direction of Sweep (Toward or Away from Outfall
5	3 Sep.	Ebb	North	Single loop made in area

Appendix G

Raw Statistical Data
Day 1

Appendix H

HP-65 Programs

HP-65 Program Form

Title: METRO - NAVIGATION PROGRAM

SWITCH TO W PRGM PRESS f PRGM TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS
LBL		
A		
DSP		
(		
(		
vg		
R/S		ENTER RA / SF-1 IF WEST f^{-1}SF1 EAST
\		
STO 1		RA^2
-		
R/S		
STO 2		RB^2
-		
CHS		
STO 3		$RB^2 - RA^2$
1		
7		
0		$2(AB + AA)$
3 3		
0 8		
-		$(RB^2 - RA^2)/2(AB+AA)$
1		
2 2		
7 5		$-\left[\dfrac{AB^2 - AA^2}{2(AB+AA)}\right]$
2 ✓		
. .		
5 ?		
÷		
STO 5		y
9 5		
7 5		AA
8 1		
0 4		
−		
↑		
X		
CHS		
RCL 1		
+		
f		
$\sqrt{x}$		
f		
TF1		
CHS		(IF WEST OF A-B BASE LINE)
NOP		

KEY ENTRY	CODE SHOWN	COMMENTS
STO 6		x
8 6		
7 2		AF
5 0		
↑		
3 3		
7 7		AC
0 0		
5 1		
STO 3		AC
÷		AF/AC ››
STO 4		SIN θ
3 3		
6 6		AD
0 4		
0 9		
↑		
RCL 3		AC
÷		AD/AC ››
STO 3		COS θ
RCL 5		
RCL 3		
X		$y \cos\theta$
RCL 6		
RCL 4		
X		$x \sin\theta$
+		$y'' = y\cos\theta + x\sin\theta$
STO 2		
RCL 6		
RCL 3		
X		$x \cos\theta$
RCL 5		
RCL 4		
X		
−		
3 3		
7 7		AC
0 0		
5 1		
−		
STO 1		$x'' = x\cos\theta - y\sin\theta - AC$
DISP		
.		
4		
R/S		RECORD x''
RCL 2		
R/S		RECORD y''

REGISTERS	
R_1	RA^2 / x''
R_2	RB^2 / y''
R_3	$RB^2 - RA^2$ / AC / $\cos\theta = AD/AC$
R_4	$\sin\theta = AF/AC$
R_5	y
R_6	x
R_7	
R_8	
R_9	

LABELS	
A	✓
B	
C	
D	
E	
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

FLAGS	
1	✓
2	

TO RECORD PROGRAM INSERT MAGNETIC CARD WITH SWITCH SET AT W/PRGM

HP-65 User Instructions

METRO - NAVIGATION PROGRAM — Page 1 of 1

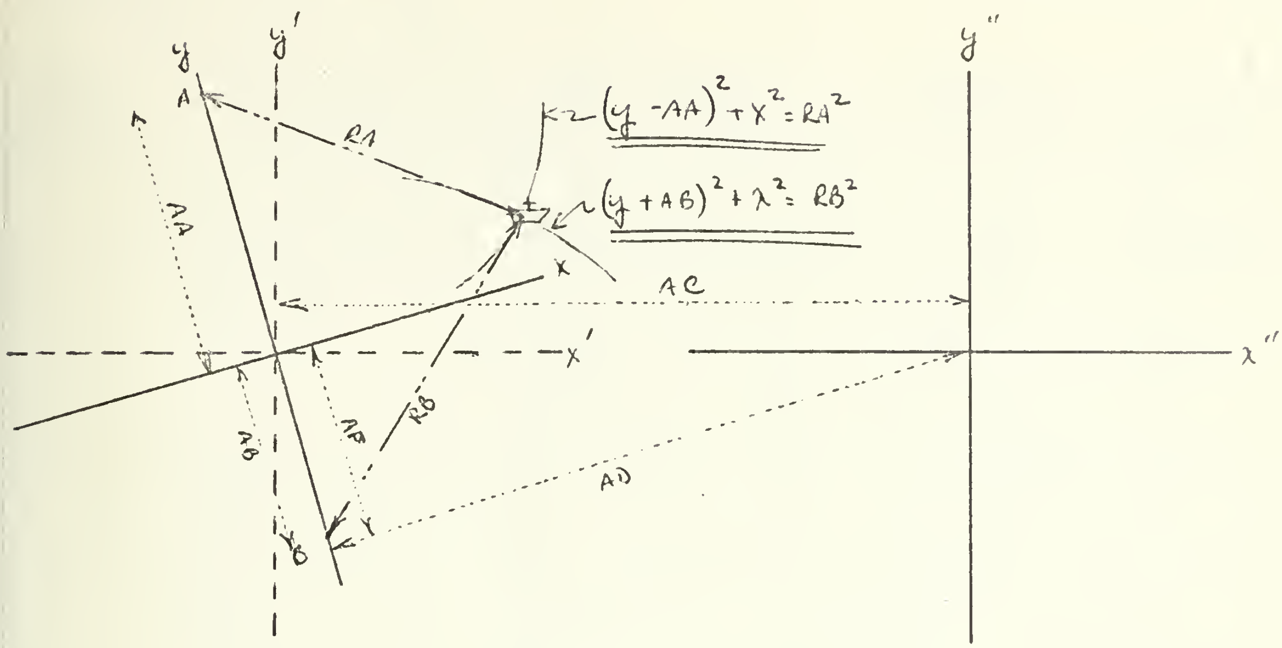
mer KARR — Date 8/21

METRO-NAVIGATION PGM

PREP.

INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
ENTER METRO-NAV. PGM				
INITIALIZE		PREP		1.
WHEN 1. APPEARS, ENTER RA	RA (METERS)	R/S		2.
WHEN 2. APPEARS, ENTER RB	RB (METERS)	R/S		X"
(RECORD X")				
AFTER RECORDING X", PUSH R/S		R/S		y"
(RECORD y")				
IF OPERATING TO THE WEST OF BASE				
INE (AND EXTENSIONS) A-B PUSH →		f	SF 1	
ONCE PUSHED, NO FURTHER ENTRY IS				
REQUIRED UNTIL RETURNING BACK TO				
THE EAST OF BASE LINE, ~~AFT~~ AT				
WHICH TIME, PUSH →		f^{-1}	SF 1	
ONCE PUSHED, NO FURTHER ENTRY				
REQUIRED WHILE TO EAST.				

METRO - NAVIGATION REDUCTION TO x'', y''. KARR, K.R. 1



$y' = y\cos\theta + x\sin\theta$ $\qquad y'' = y'$

$x' = x\cos\theta - y\sin\theta$ $\qquad x'' = x' - AC$

$$-\left[(y-AA)^2 + x^2 = RA^2\right]$$

$$(y+AB)^2 + x^2 = RB^2$$

$$(y+AB)^2 - (y-AA)^2 = RB^2 - RA^2$$

$$y^2 + 2ABy + AB^2 - y^2 + 2AAy - AA^2 = RB^2 - RA^2$$

$$2y\,(AB + AA) = RB^2 - RA^2 - AB^2 + AA^2$$

$$y = \frac{RB^2 - RA^2 - (AB^2 - AA^2)}{2(AB + AA)}$$

$$x = \left\{RA^2 - \left[\frac{RB^2 - RA^2 - (AB^2 - AA^2)}{2(AB + AA)} - AA\right]^2\right\}^{1/2}$$

$$\underline{y'' = y\cos\theta + x\sin\theta}$$

$$\underline{x'' = x\cos\theta - y\sin\theta - AC}$$

HP-65 Program Form

Title: Centroids (Dye Data), $\bar{X}$, $\bar{Y}$ Page 1 of 1

SWITCH TO W/PRGM PRESS f PRGM TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS
LBL		
/		
STO 1		
R/S		
LBL		
STO 2		
R/S		
LBL		
.		
(		
)		
g x≤y		
f		
1^x		
STO 3		
STO		
+		
RCL 1		
x		
STO		
5		
RCL 3		
RCL 2		
x		
STO		
+		
6		
RCL 1		
1		
+		
STO 1		
R/S		
RCL 2		
1		
+		
STO 2		
R/S		
LBL		
D		
RCL 5		
RCL 4		

KEY ENTRY	CODE SHOWN	COMMENTS
÷		
R/S		
RCL 6		
RCL 4		
÷		
R/S		
LBL		
E		
↑		
3		
−		
.		
1		
x		
1		
0		
g x≷y		
g		
y^x		
CHS		
STO 3		
STO		
+		
4		
RCL 1		
CHS		
x		
STO		
+		
5		
RCL 3		
RCL 2		
CHS		
x		
STO		
+		
6		
RCL 1		
R/S		

REGISTERS	
R_1	X
R_2	y
R_3	c_i
R_4	Σc_i
R_5	$\Sigma c_i x$
R_6	$\Sigma c_i y$
R_7	
R_8	
R_9	

LABELS	
A	✓
B	✓
C	✓
D	✓
E	✓
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

FLAGS	
1	
2	

HP-65 User Instructions

Centroids, $\bar{X}$, $\bar{Y}$, Dye Data — Page 1 of 1

[Progra]mmer: K. KARR — Date: 1/15/75

$\bar{X}$; $\bar{Y}$ Dye Data

X	Y	C	CAL	ERROR

INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
Feed pgm strip				
Enter initial X		X		X
Enter initial Y		Y		Y
" " concentration for X, Y		C		next X will appear
Repeat 4 for next X				
when at end of row,		R/S		new Y
Enter new X		X		new X
Repeat 4				
Compute $\bar{X}$, $\bar{Y}$ when complete		CAL		$\bar{X}$
		R/S		$\bar{Y}$
		R/S		ΣC
Clears for next set of data.		R/S		0.00
If Error made, enter X, Y of error		X	Y	
Re-insert erroneous entry concentration		ERROR		
Insert X, Y for new, correct entry		X	Y	
Enter correct conc.		C		next X will appear

HEWLETT hp PACKARD

HP-65 User Instructions

σ_x^2 σ_x σ_y^2 σ_y Dye Data — Page 1 of 1

mer K. Karr — Date 1/15/75

σ_x^2	σ_x	σ_y^2	σ_y	
X	Y	C	Comp.	

INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
Enter Pgm Strip				
		C		—
hen pgm stops, switch to "write pgm" then →		g	DEL	
Enter $\bar{X}$ (eg: 13.2 : to ~~one~~ decimal place)				
switch to "run"		R/S		—
hen pgm stops, switch to "write pgm" then		g	DEL	
Enter $\bar{Y}$ (eg: 9.7 : to one decimal place)				
switch to "run".		R/S		
CLEAR REGISTERS		f	reg	
Enter initial X value		X		~~next~~ X will appear
Enter initial Y value		Y		Y will appear
Enter concentration for X, Y		C		next X will appear
when ready for new Y following C entry		R/S		new Y will appear
Enter new X & repeat step 12		X		X will appear
when data processed (record)		Comp.		σ_x^2
" "		R/S		σ_x
" "		R/S		σ_y^2
" "		R/S		σ_y
" "		R/S		ΣC_i
To start anew, turn off calculator,				
eprogram, & repeat above.				

HEWLETT hp PACKARD

HP-65 Program Form

Title: σ_x^2, σ_x, σ_y^2, σ_y dye data. Page 1 of 1

SWITCH TO W/PRGM PRESS [f] [PRGM] TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS
LBL		
A		
STO 1		
R/S		
LBL		
(		
STO 2		
R/S		
LBL		
C		
/		
:		
-		
g x≷y		
x		
T 1		
R/S		
g NOP		
STO 3		
TO		
+		
RCL 1		
R/S		
-		
`		
RCL 3		
(		
STO		
-		
3		
RCL		
2		
R/S		
-		
↑		
x		
RCL 3		
x		
STO		
+		
9		
RCL 1		

KEY ENTRY	CODE SHOWN	COMMENTS
1		
+		
STO 1		
R/S		
RCL 2		
1		
+		
STO 2		
R/S		
LBL		
D		
RCL 8		
RCL 4		
÷		
R/S		$\Sigma c_i(x-\bar{x})^2/\Sigma c_i = \sigma_x^2$
f		
$\sqrt{x}$		
R/S		$\sqrt{\Sigma c_i(x-\bar{x})^2/\Sigma c_i} = \sigma_x$
RCL 9		
RCL 4		
÷		
R/S		$\Sigma c_i(y-\bar{y})^2/\Sigma c_i = \sigma_y^2$
f		
$\sqrt{x}$		
R/S		$\sqrt{\Sigma c_i(y-\bar{y})^2/\Sigma c_i} = \sigma_y$
RCL		
4		
R/S		Σc_i
80		
90		
100		

REGISTERS	
R_1	x_i
R_2	y_i
R_3	c_i
R_4	Σc_i
R_5	-
R_6	-
R_7	-
R_8	$\Sigma c_i(x-\bar{x})^2$
R_9	$\Sigma c_i(y-\bar{y})^2$

LABELS	
A	✓
B	✓
C	✓
D	✓
E	
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

FLAGS	
1	✓
2	

TO RECORD PROGRAM INSERT MAGNETIC CARD WITH SWITCH SET AT W/PRGM

HP-65 Program Form

Title: TWO BEARING FIX — Page 1 of 1

SWITCH TO W/PRGM PRESS f PRGM TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS	KEY ENTRY	CODE SHOWN	COMMENTS
LBL			–		
/			STO 7		
.			x		
R/S			RCL 8		
ST 4			+		
			STO 8		
R/S			RCL 7		
S 5			RCL		
			9		
R/S			60 ÷		
S 6			STO 7		
			R/S		
T 1			RCL 8		
G 0			R/S		
			LBL		
LBL			B		
			f		
			SF 1		
)			GTO		
RCL 6			70 A		
:			LBL		
-			1		
TN			4		
S 7			R/S		
			STO 1		
)			5		
RCL 3			R/S		
.			STO 2		
-			6		
TN			80 R/S		
S 8			STO 3		
-			f^{-1}		
S			SF 1		
			GTO		
STX			2		
RCL					
9					
÷					
RCL 2					
RCL 8			90		
RCL 1					
X					
–					
STO 8					
RCL 7					
RCL 4					
X					
CHS					
RCL 5					
+			100		

REGISTERS

- R_1: x_{s_2}
- R_2: y_{s_2}
- R_3: B_{T_2}
- R_4: x_{s_1}
- R_5: y_{s_1}
- R_6: B_{T_1}
- R_7: a_1, $b_2 - b_1$, x
- R_8: a_2, b_2, y
- R_9: $a_1 - a_2$

LABELS

- A ✓
- B ✓
- C
- D
- E
- 0
- 1 ✓
- 2 ✓
- 3
- 4
- 5
- 6
- 7
- 8
- 9

FLAGS

- 1 ✓
- 2

TO RECORD PROGRAM INSERT MAGNETIC CARD WITH SWITCH SET AT W/PRGM

HP-65 User Instructions

TWO BEARING FIX — Page 1 of 1

[Program]mer K. KARR — Date 8/26/74

TWO BEARING FIX

1 SET | 2 SETS

INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
ENTER PROGRAM				
IF USING 1 SET OF DATA (X_S, Y_S, B_T)				
[I]N MACH. FROM PREVIOUS CALC., PUSH		1 SET		1
WHEN 1 APPEARS ENTER	X_S	R/S		2
WHEN 2 APPEARS ENTER	Y_S	R/S		3
WHEN 3 APPEARS ENTER	B_T	R/S		X_0
RECORD X_0, then PUSH		R/S		Y_0
RECORD Y_0,				
If using 2 SETS OF DATA (X_{1S}, Y_{1S}, B_{T_1}				
and X_{2S}, Y_{2S}, B_{T_2}) external of calculator				
push		2 SETS		1
When 1 appears, enter	X_{S_1}	R/S		2
" 2 " "	Y_{S_1}	R/S		3
" 3 " "	B_{T_1}	R/S		4
" 4 " "	X_{S_2}	R/S		5
" 5 " "	Y_{S_2}	R/S		6
" 6 " "	B_{T_2}			X_0
Record X_0 then Push		R/S		Y_0
Record Y_0				

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HP-65 User Instructions

SEXTANT CONVERSION TO TRUE BEARING — Page 1 of 1

er K. KARR — Date 8/26/74

SEXTANT CONV. TO TRUE BEARING

PREP | SEC. INT. | | |

INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
ENTER PROGRAM				
INITIALIZE		PREP		
WHEN 1 APPEARS, ENTER X_r	(x_r meters)	R/S		
When 2 APPEARS, ENTER Y_r	(y_r meters)	R/S		
WHEN 3 APPEARS, ENTER SEXTANT	deg. : min.			
BEARING. IF CW FROM REF. OBJECT	" "	R/S		B_T
IF CCW FROM REF. OBJ.		CHS	R/S	B_T
(round entry to nearest minute)				
RECORD TRUE BEARING * (B_T)				
* IF B_T is > 360°, PUSH		360	−	B_T

HP-65 Program Form

Sextant Conversion to True Bearing Page 1 of 1

(S)WITCH TO W/PRGM PRESS f PRGM TO CLEAR MEMORY.

KEY ENTRY	CODE SHOWN	COMMENTS	KEY ENTRY	CODE SHOWN	COMMENTS
LC			g(x=y)		
/			GTO		
			3		
R5			1		
T 3			−		
/			STO 8		
R5			RCL3		
4			CHS		
L2			STO 3		
			RCL7		
5			GTO		
C3			2		
C1			LBL		
			3		
T6			RCL 8		
			3		
T8			g x>y		
			2		
T 3			−		
RL5			70 9		
			0		
			×		
x y			RCL5		
G0			RCL6		
			÷		
L6			f^{-1}		
)			tan		
c			g		
x y			ABS		
			80 RCL3		
			×		
			+		
			STO 5		
S07			3		
G0			R/S		
			LBL		
LL			B		
			f^{-1}		
RL6			→D.MS.		
0			90 RCL5		
g x>y			+		
			0		
			g x>y		
			3		
			6		
S07			0		
G02			+		
LL			+		
			STO 3		
RL8			100 R/S		

REGISTERS

- R_1 X_s
- R_2 y_s
- R_3 X_r; $1 (\pm)$; B_T
- R_4 y_r
- R_5 $y_r - y_s$; $(\pm)\tan^{-1}\left|\frac{y_r-y_s}{X_r-X_s}\right|$
- R_6 $X_r - X_s$
- R_7 1, 2, 3 or 4
- R_8 4
- R_9

LABELS

- A ✓
- B ✓
- C
- D
- E
- 0
- 1 ✓
- 2 ✓
- 3 ✓
- 4
- 5
- 6
- 7
- 8
- 9

FLAGS

- 1
- 2

TO RECORD PROGRAM INSERT MAGNETIC CARD WITH SWITCH SET AT W/PRGM

Thesis 159529
K14478 Karr
Spatial and temporal behavior of the West Point sewage effluent in Puget Sound.